Development of Data-Driven Approach based on the Empirical Interpolation Method for Thermal-Hydraulics Analysis

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GOAL

Integrate Numerical Modelling and Experimental Data

CHALLENGES

1. Select Best Measurement Points
2. Reconstruct Spatial Field from Local Measures
3. Perform Parametric State Estimation Based on Measures
4. Reconstruct Spatial Field from Parameters Estimation

Full Order Solutions are approximated combining specialized basis functions $\varphi_i(\boldsymbol{x})$ derived from a Train Set of Solutions.

$$g(\boldsymbol{x} | \boldsymbol{\mu}) \approx \sum \beta_i(\boldsymbol{\mu}) \varphi_i(\boldsymbol{x})$$

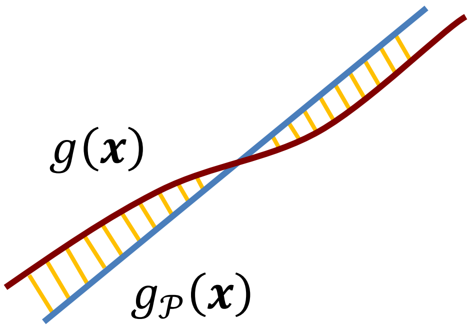
online offline

The spatial and parametric dependence are separated. Only a few coefficients $\beta_i(\boldsymbol{\mu})$ need to be online estimated relying on models or experimental data.

models → Projection

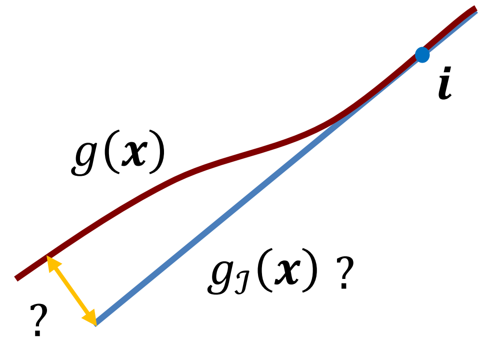
experimental data → Interpolation

Projection
(needs a model)



PRJ condition : $\min(\|g - g_{\mathcal{P}}\|_{\chi})$

Interpolation
(needs measures)



INT condition : $g(\boldsymbol{i}) = g_{\mathcal{I}}(\boldsymbol{i})$

↓

Using proper methods, providing proper train sets, reconstruction error goes to zero for $s \to \infty$

Projection Based Methods

GREEDY

POD

Optimal!

Scalar Fields Projection? yes

Vector Fields Projection? yes

Interpolation Based Methods

EIM

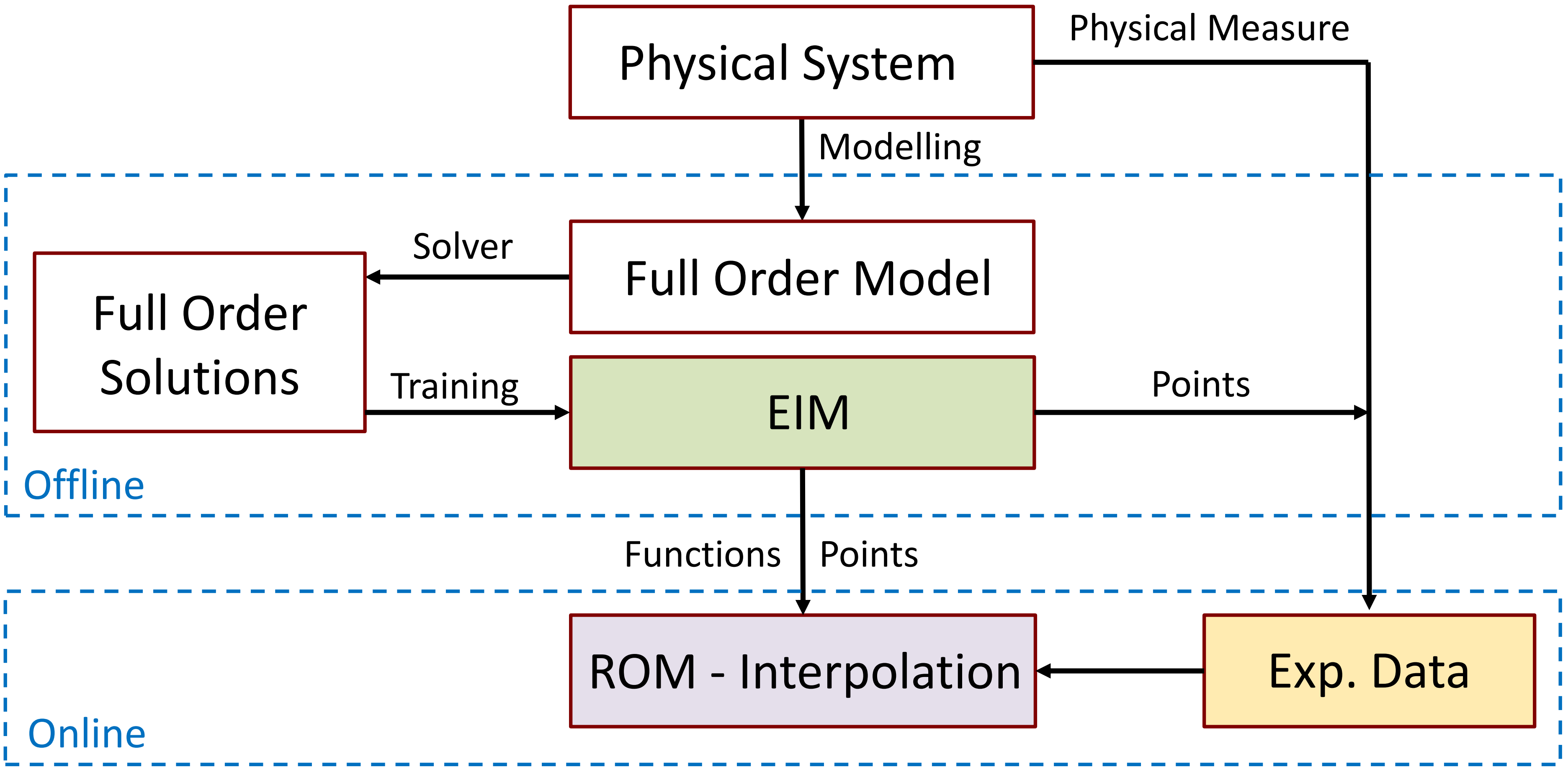
GEIM

Scalar Fields Interpolation? yes

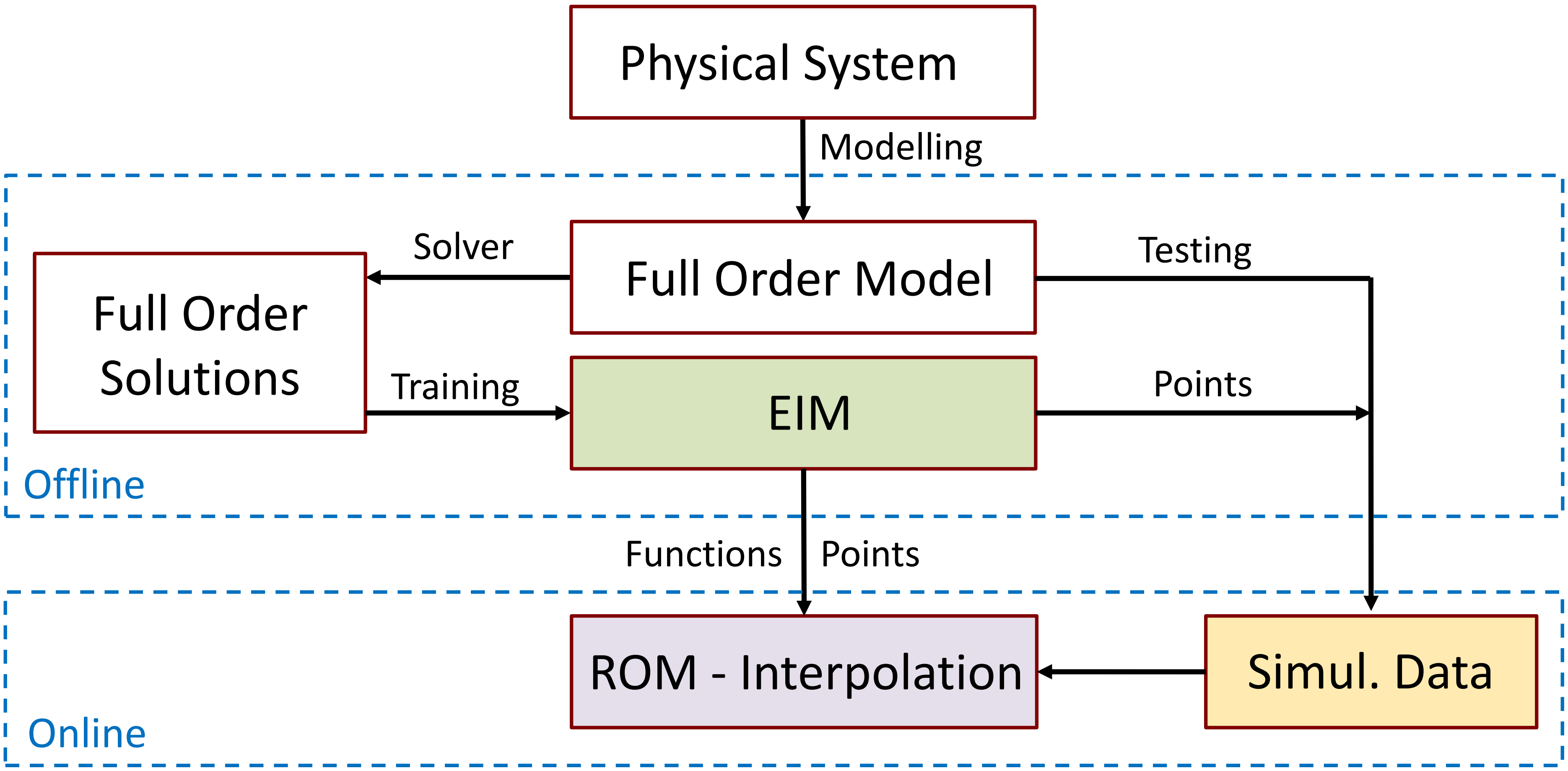
Vector Fields Interpolation? no

New!

Data Driven Model Order Reduction (Experimental)



Data Driven Model Order Reduction (Numerical)



EIM : Greedy Basis and Points Selection

$g(\boldsymbol{x};\boldsymbol{\mu})$ = Train Set Solutions

$$\mathcal{J}^{(s)}_{\{\emptyset\}} g(\boldsymbol{x};\boldsymbol{\mu}) = 0$$

$s = s + 1$

Interpolated Fields ← $\mathcal{J}^{(s)}_{\{(\rho,\boldsymbol{i})_j\}} g(\boldsymbol{x};\boldsymbol{\mu})$

↓

Residuals Construction ← $r(\boldsymbol{x};\boldsymbol{\mu}) = g(\boldsymbol{x};\boldsymbol{\mu}) - \mathcal{J}^{(s)}_{\{(\rho,\boldsymbol{i})_j\}} g(\boldsymbol{x};\boldsymbol{\mu})$

↓

Parameter Selection ← $\boldsymbol{\mu}^{s+1} = \underset{\boldsymbol{\mu}\in\mathcal{D}}{\operatorname{argmax}} \|r(\boldsymbol{x};\boldsymbol{\mu})\|_{\chi}$

↓

New Measure Point ← $\boldsymbol{i}^{s+1} = \underset{\boldsymbol{x}\in\Omega}{\operatorname{argmax}} |r(\boldsymbol{x};\boldsymbol{\mu}^{s+1})|$

↓

New Basis Function ← $\rho_{s+1}(\boldsymbol{x}) = r(\boldsymbol{x})/|r(\boldsymbol{i}^{s+1})|$

$e_{\mathcal{J},L^\infty} \to 0$ proved!

EIM : Scalar Field Interpolation

EIM performs the interpolation of physical fields looking for a linear combination of the basis functions compatible with local measures.

Scalar: T & ρ_i

$$T(\boldsymbol{r}, t; \mu) \approx \sum_{i=1}^{s} b_i(t, \boldsymbol{\mu}) \cdot \rho_i(\boldsymbol{r})$$

Scalars: b_i

$$\begin{bmatrix} b_1(\boldsymbol{\mu}) \\ \vdots \\ b_s(\boldsymbol{\mu}) \end{bmatrix} = \begin{bmatrix} \rho_1(\boldsymbol{t}^1) & \cdots & \rho_M(\boldsymbol{t}^1) \\ \vdots & \ddots & \vdots \\ \rho_1(\boldsymbol{t}^s) & \cdots & \rho_M(\boldsymbol{t}^s) \end{bmatrix}^{-1} \cdot \begin{bmatrix} T(\boldsymbol{t}^1; \boldsymbol{\mu}) \\ \vdots \\ T(\boldsymbol{t}^s; \boldsymbol{\mu}) \end{bmatrix}$$

Low triangular square matrices

EIM : Vector Field Interpolation (New!)

The same problem becomes much harder when vector fields are involved!

Vectors $\boldsymbol{u}$ & $\boldsymbol{\rho}_{\mathrm{i}}$

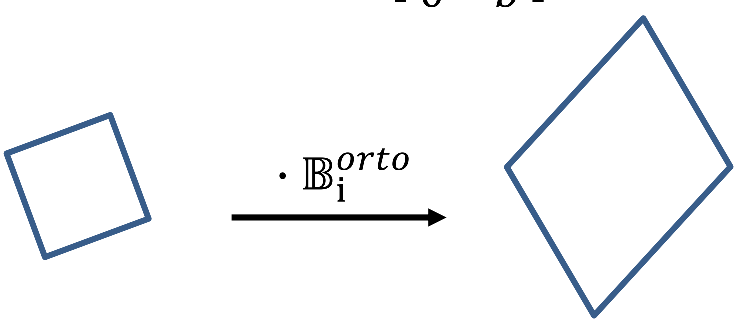
$$\boldsymbol{u}(\boldsymbol{r}, t; \mu) \approx \sum_{\mathrm{i}=1}^{s} \mathbb{B}_{\mathrm{i}}\,(t, \boldsymbol{\mu}) \cdot \boldsymbol{\rho}_{\mathrm{i}}(\boldsymbol{r})$$

$\mathbb{B}_{\mathrm{i}}$ Matrices?

?

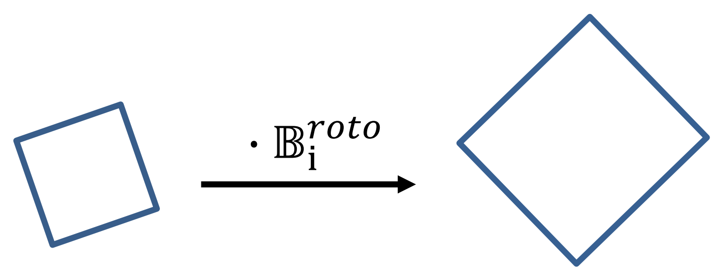
orto

$$\mathbb{B}_{\mathrm{i}}^{orto} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

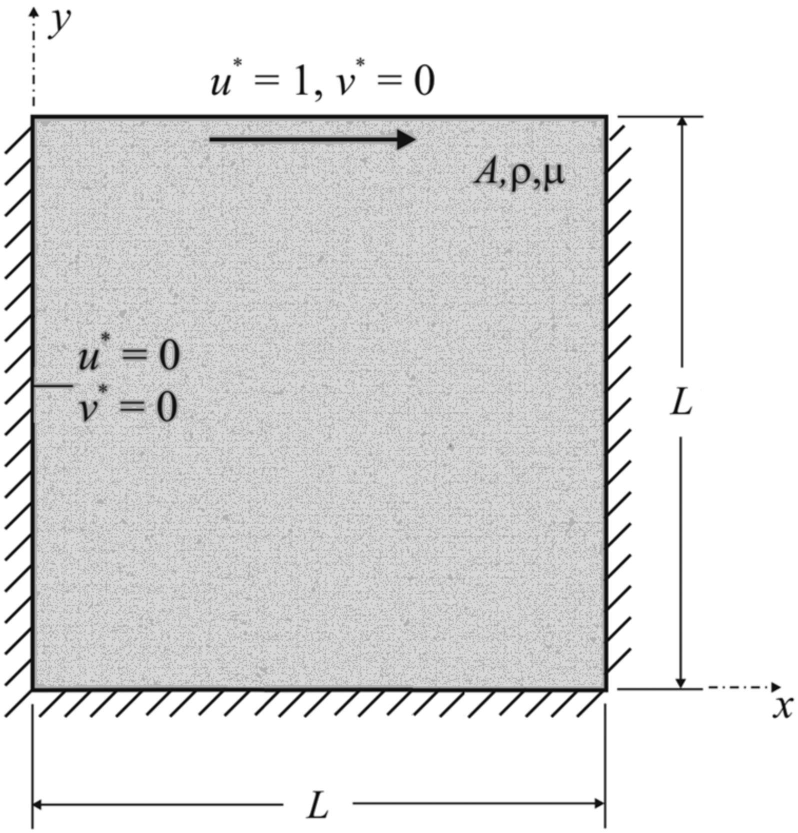


roto

$$\mathbb{B}_{\mathrm{i}}^{roto} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$



Lid-Driven Cavity Benchmark



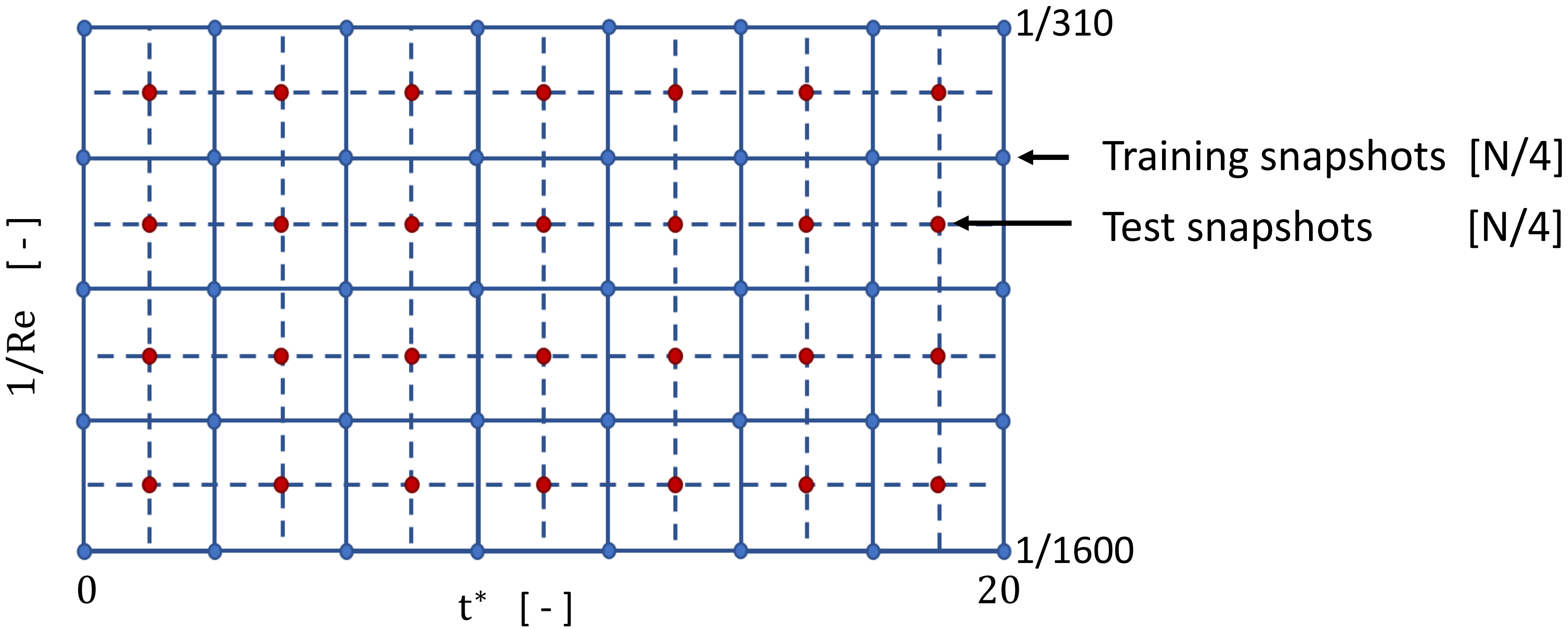
Navier-Stokes problem in the unit square:
$\Omega \coloneqq [0,1]^2, \mathbf{u}^*(t^* = 0) = 0.$

$$\begin{cases} \nabla^* \cdot \mathbf{u}^* = 0 \\ \dfrac{\partial \mathbf{u}^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla^*)\mathbf{u}^* - \dfrac{1}{\mathrm{Re}} \Delta^* \mathbf{u}^* + \nabla p^* = 0 \end{cases}$$

Problem solved in a given time interval, for 1/Re in a given range with OpenFoam (FV code):

- 1/Re in [1/1600, 1/310] ⟵ Laminar
- t^* in [0, 20]

Parameters Discretization

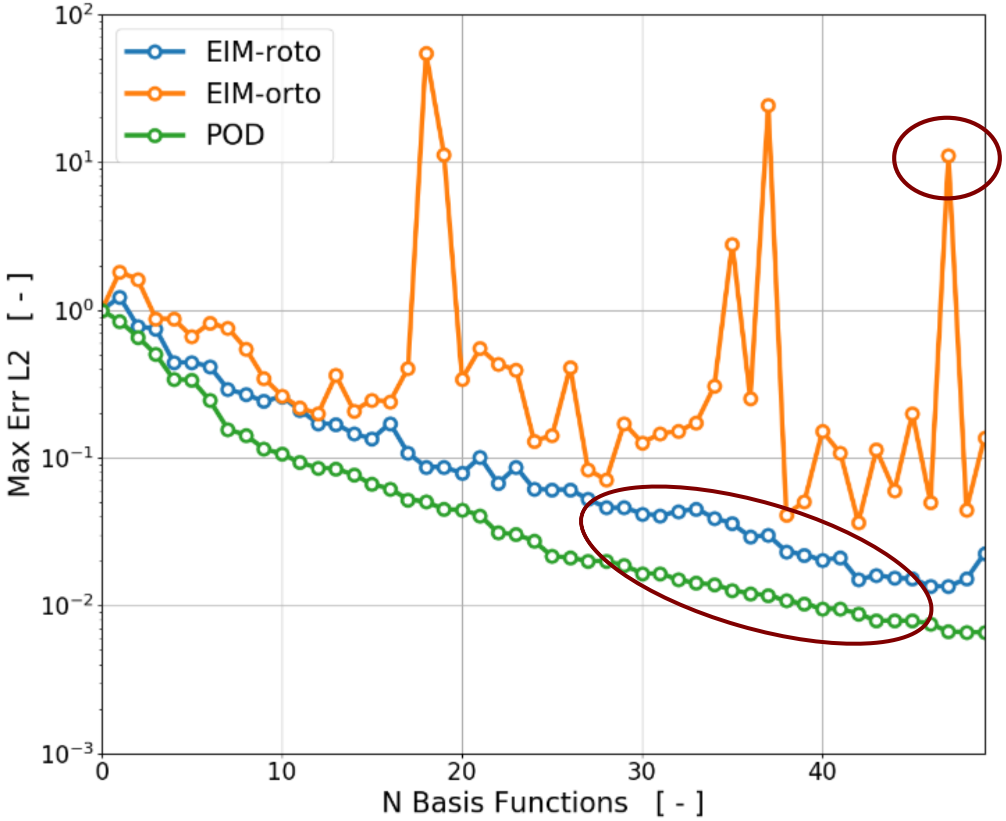


$1/\mathrm{Re} \coloneqq [1 : 0{,}16 : 5{,}16]/1600$ ← 27 points [14+13]

$t^* \coloneqq [0.08 : 0{,}08 : 20]$ ← 250 points [125+125]

$N = 6750$

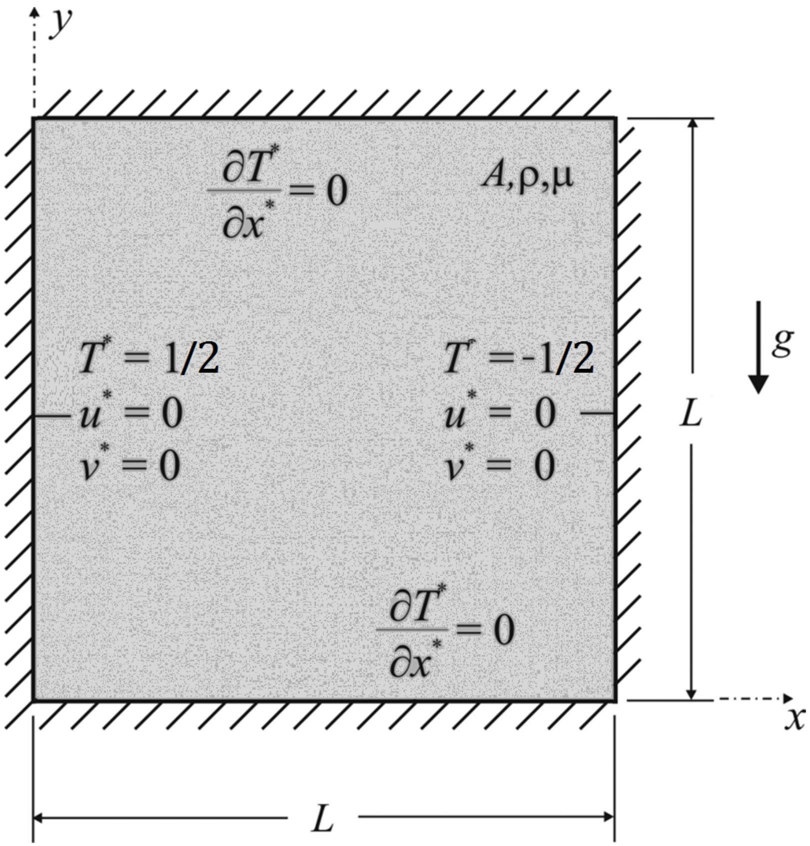
max error L^2 for EIM–orto VS EIM-roto VS POD



The EIM-orto is unable to effectively control the error.

Interpolation and EIM reduced spaces allow nearly optimal performances.

Buoyancy-Driven Cavity Benchmark



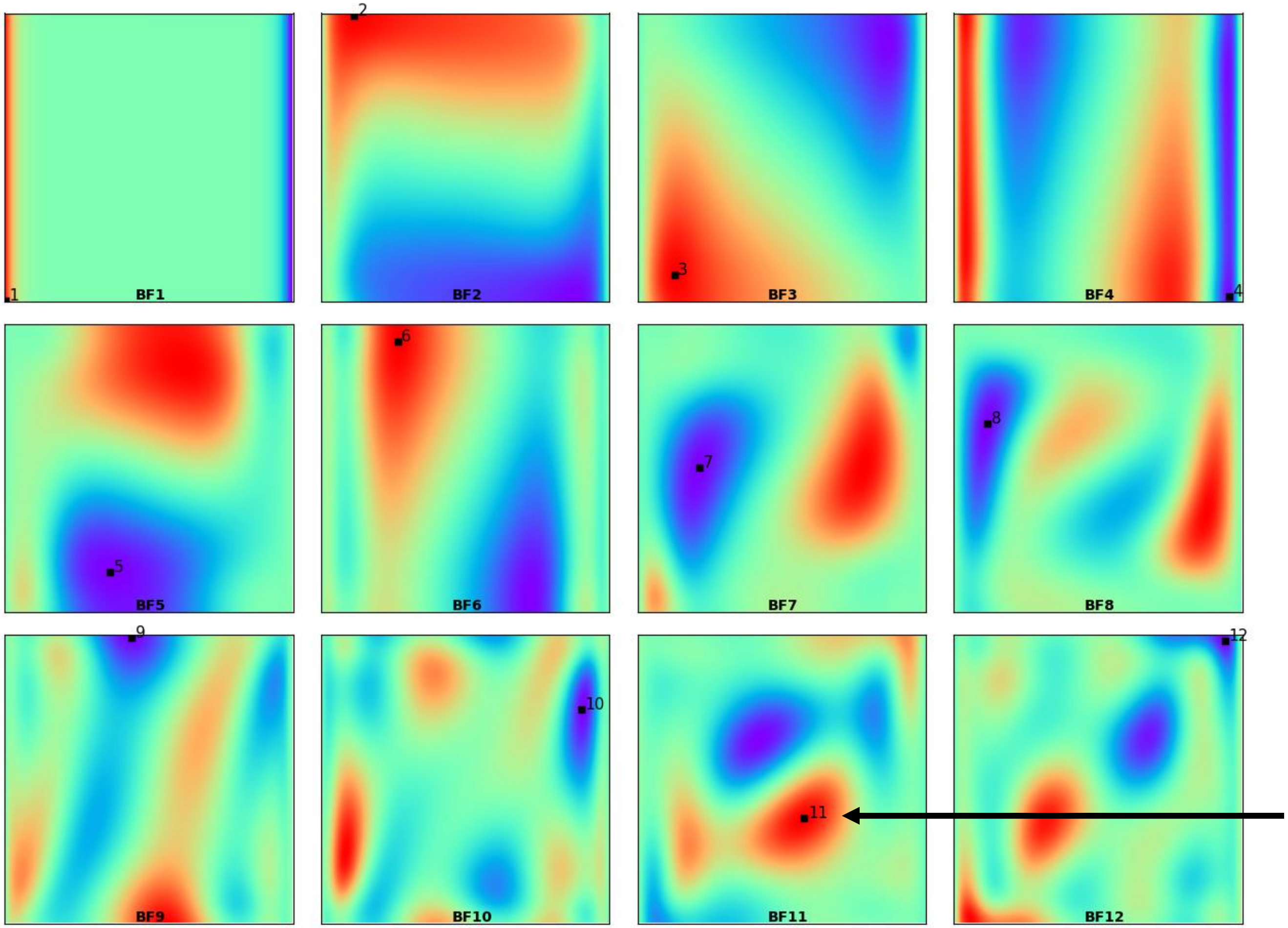
Under Boussinesq approximation momentum and heat equations are coupled.

$$\begin{cases} \nabla^* \cdot \mathbf{u}^* = 0 \\ \frac{\partial \mathbf{u}^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla^*)\mathbf{u}^* - \Delta^* \mathbf{u}^* + \nabla p^* + \mathrm{Gr} \cdot T^* \boldsymbol{e}_y = 0 \\ \frac{\partial T^*}{\partial t^*} + (\mathbf{u}^* \cdot \nabla^*) T^* - \mathrm{Pr}^{-1} \cdot \Delta^* T^* = 0 \end{cases}$$

Problem solved in a given time interval, with Gr in a specific range, using OpenFoam (FV code):

- Gr in [4’000, 100’000] ⟵ Laminar
- $\mathrm{Pr} = 1$
- t^* in [0, 0.25]

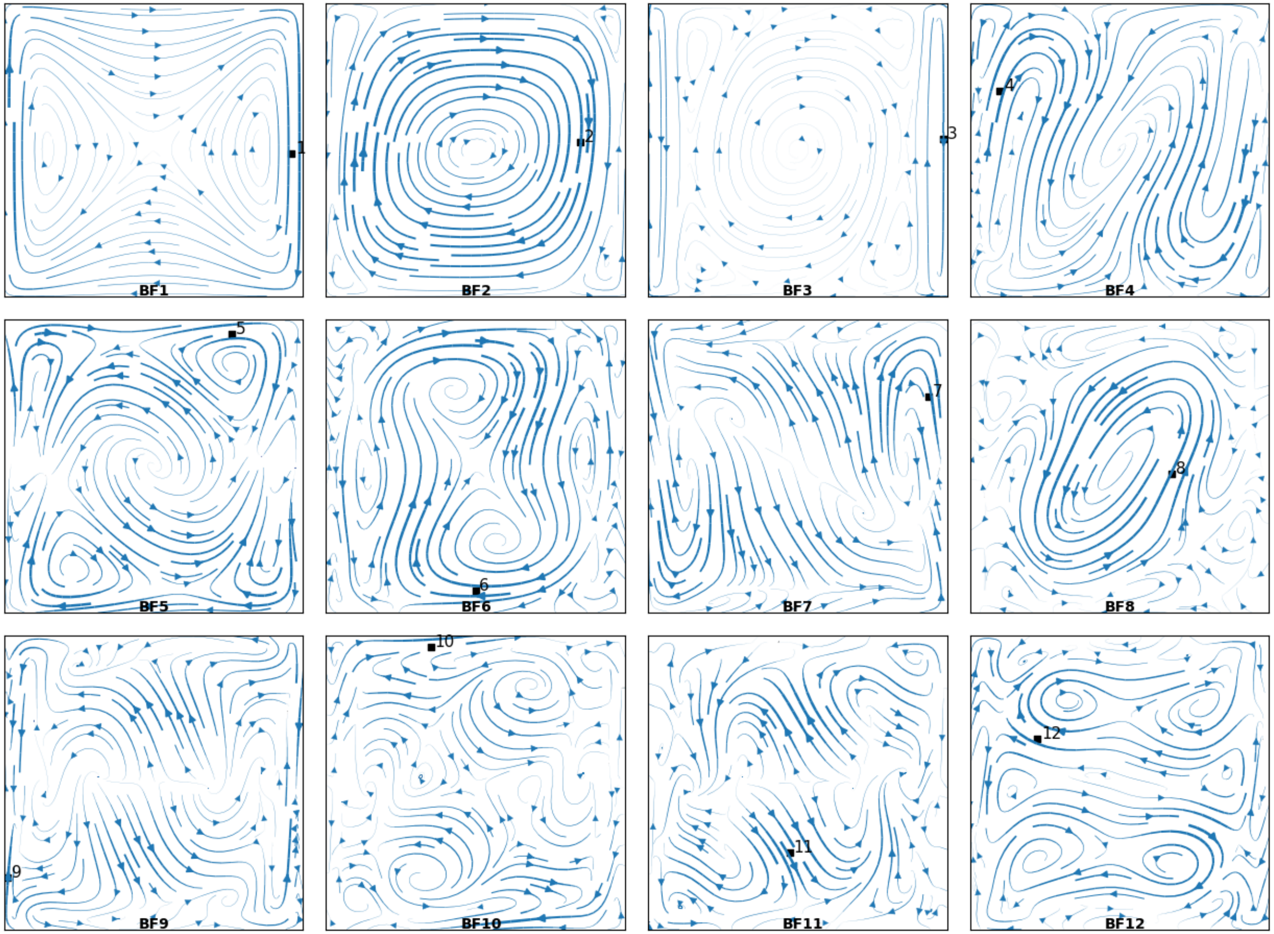
Basis Functions – EIM (T)



The BFs information content tends to be localized.
Information is used near where it is collected.

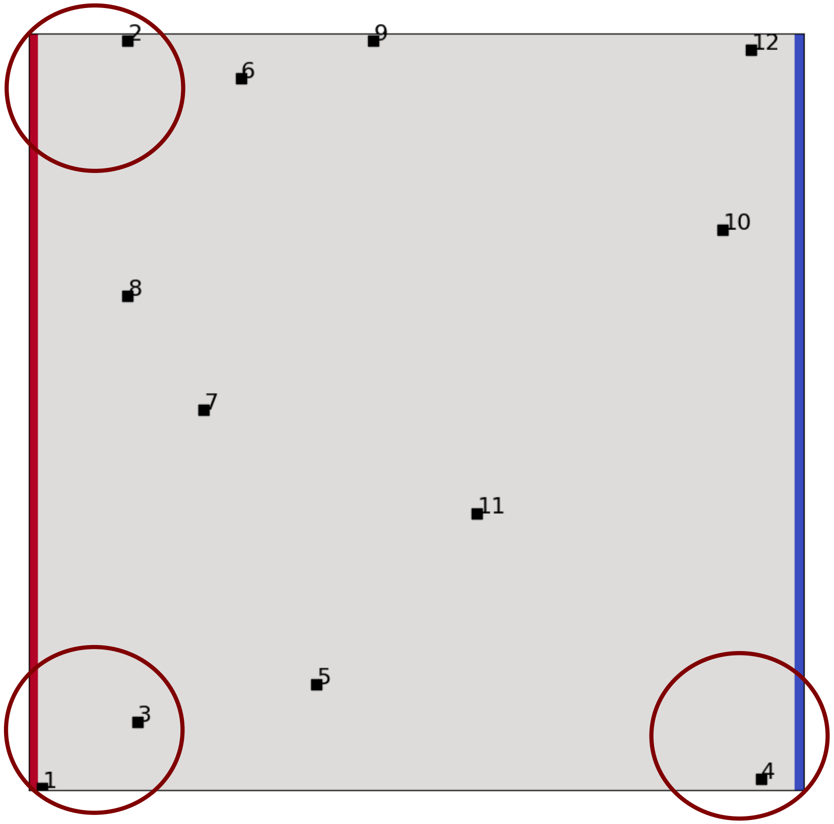
By construction the interpolation points are set in points of maxima magnitude.

Basis Functions – EIM (u)



Internal point-symmetry are conserved and turns into BFs properties.

Interpolation Points – EIM (T)

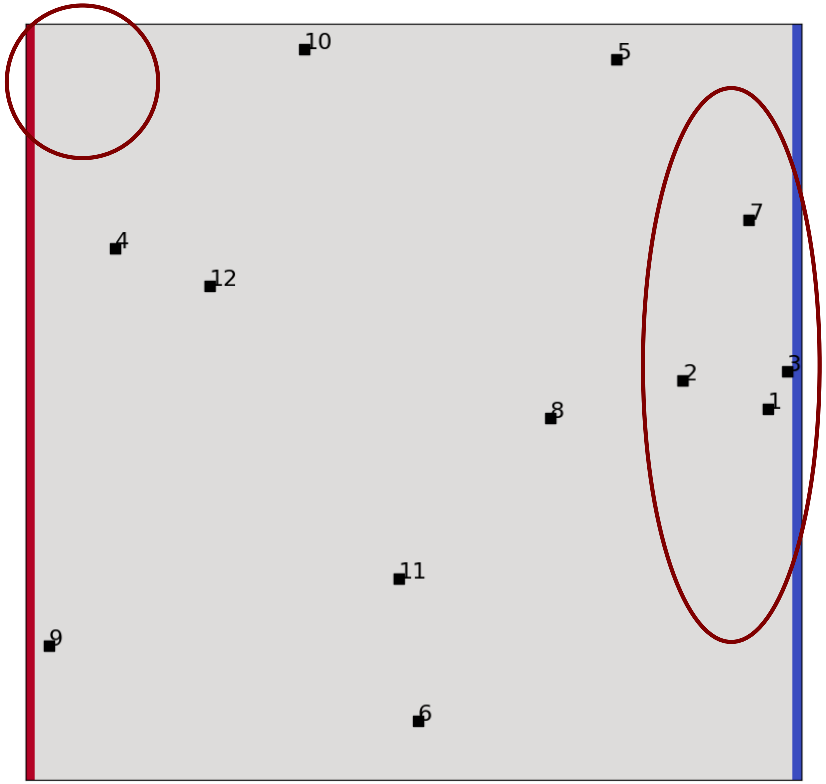


Lower order interpolation points are higher importance measurement positions.

The first points are located on the edges, where most significant non linear effects are observed.

Interpolation Points – EIM (u)

On the edges, where viscous effects are maximum, no useful information is provided. No interpolation points are here located.

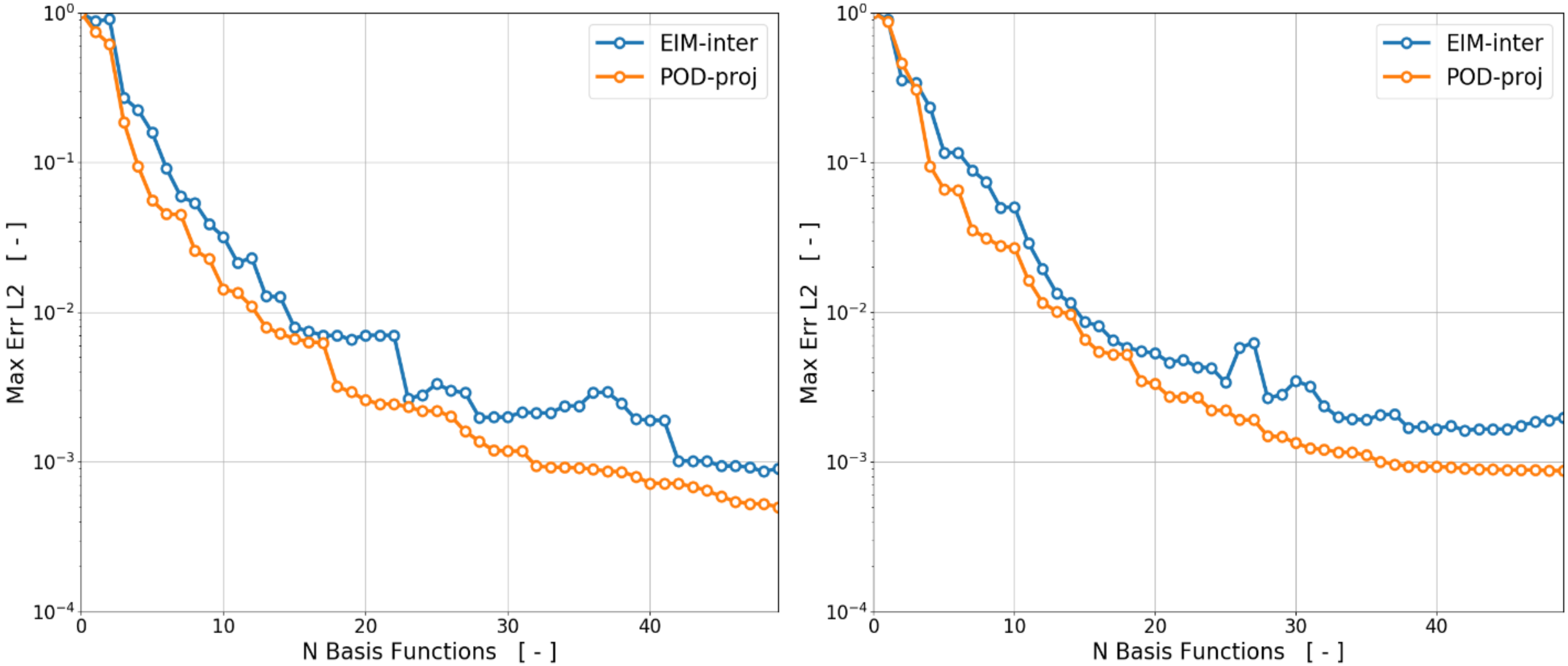


The first interpolation points are located on the vertical walls. The velocity profile is here closely correlated to the flow regime.

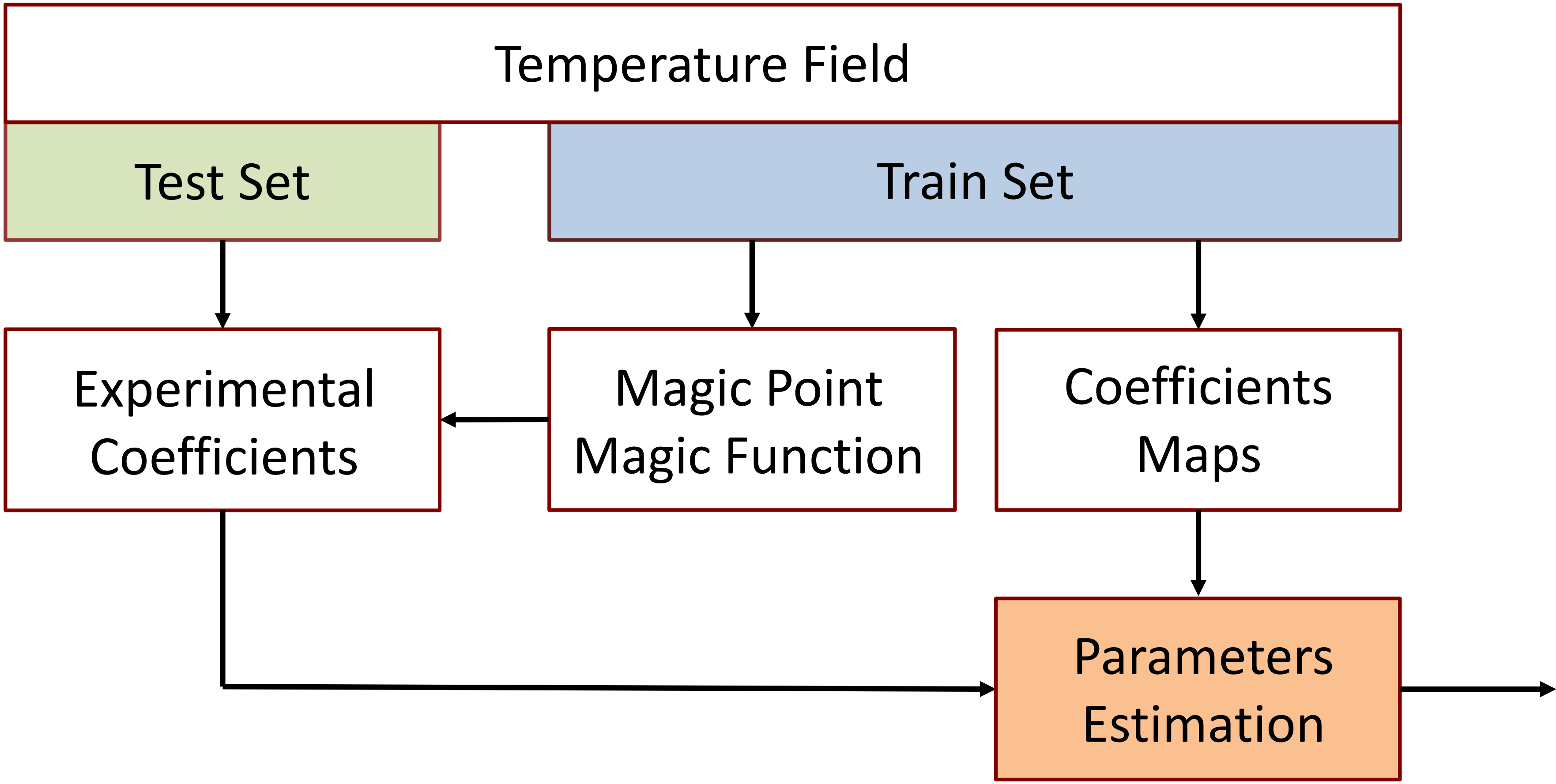
A Posteriori Performance Analysis

max error L^2 for EIM - T

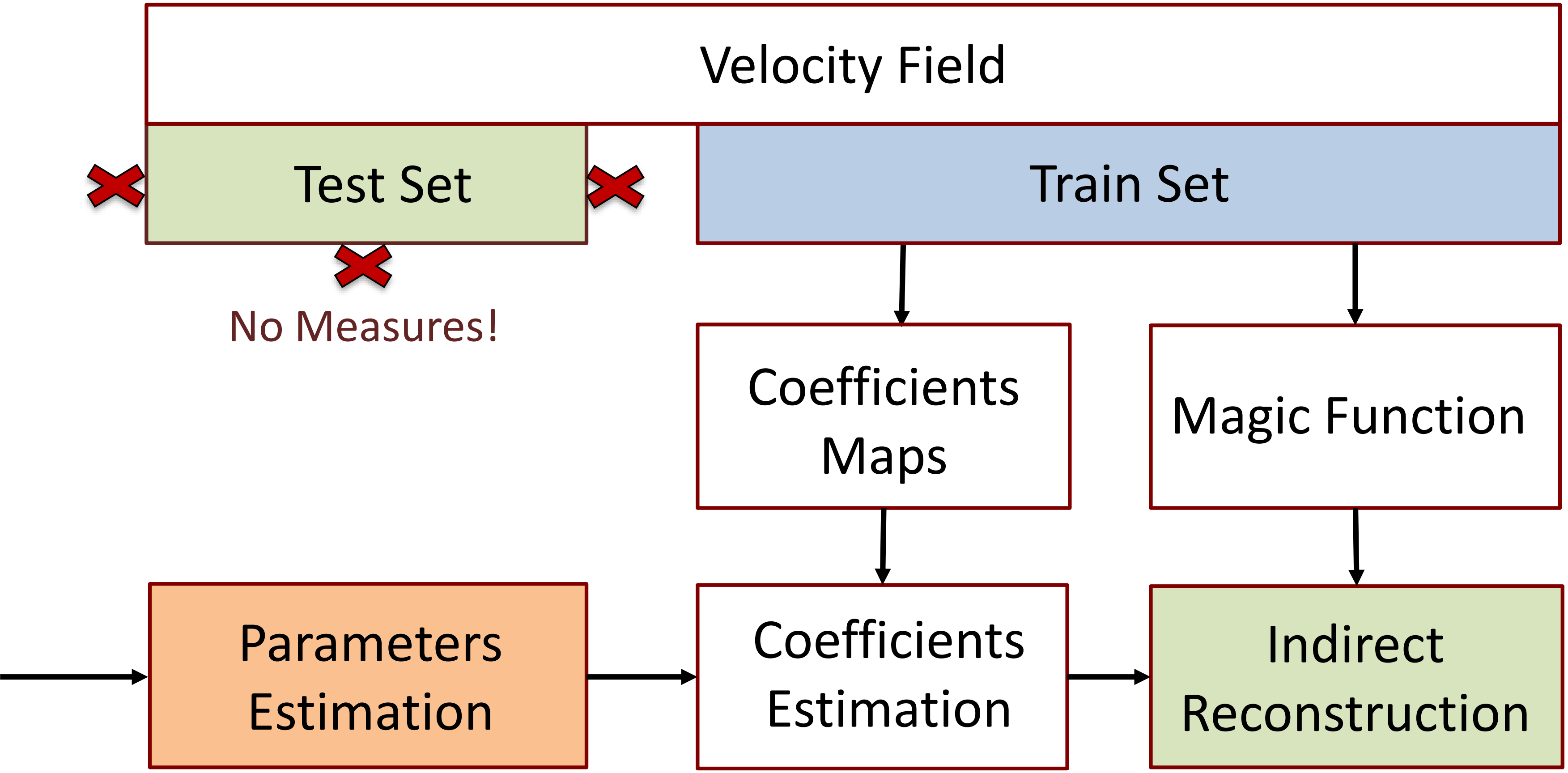
max error L^2 for EIM – U



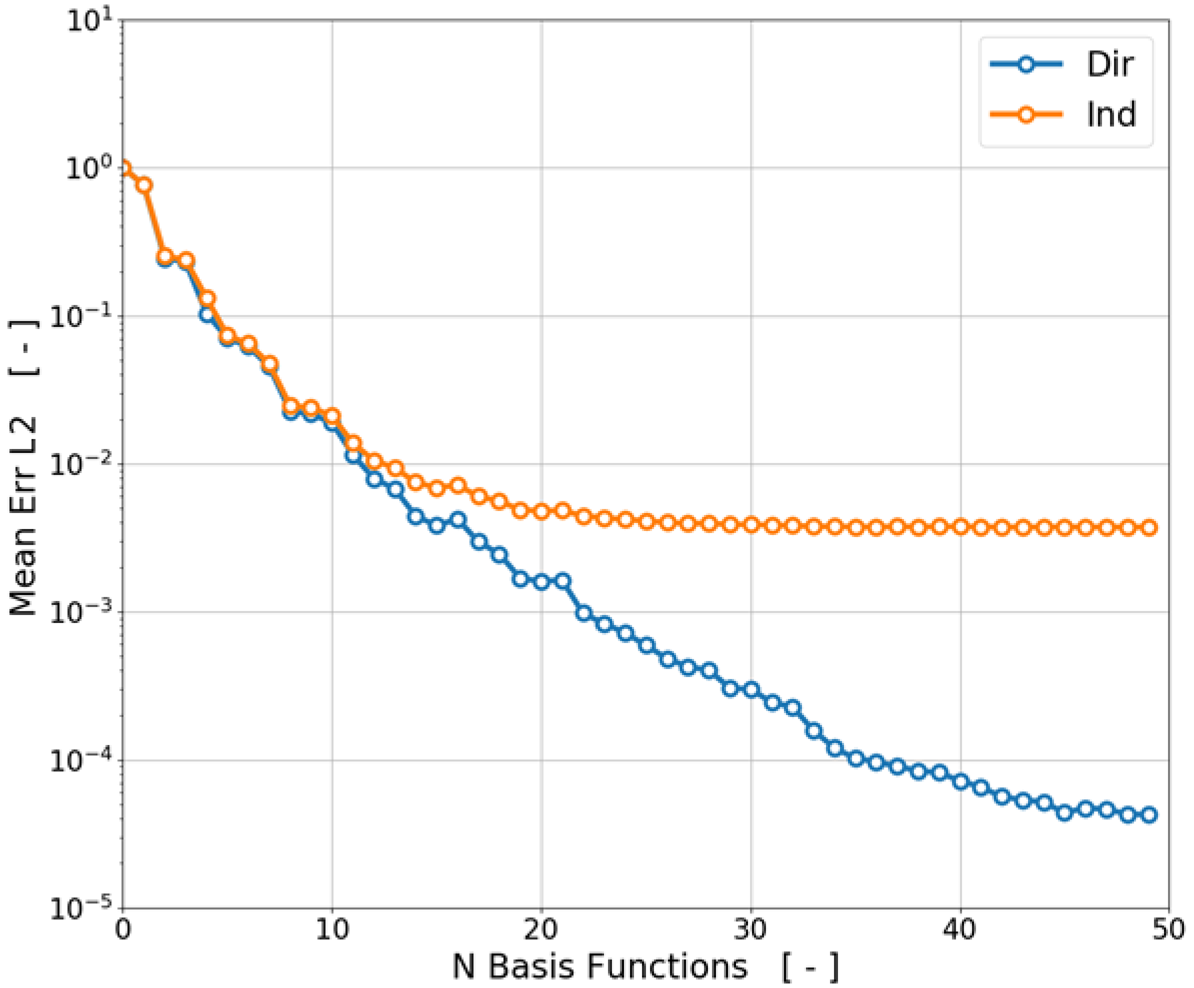
Parameters Estimation (New!)



Indirect Reconstruction (New!)



Indirect Reconstruction Mean Error L^2



Due to the low regularity of the high order coefficients maps, a good estimation of the high order terms is impossible.

Anyway, with less then 15 BFs we got an average indirect reconstruction error lower then 1%

CHALLENGES AND RESULTS

1. Good Measure Points Selection → Physically Consistent
2. Accurate Direct Interpolations → Error below 1%
3. Accurate Indirect Reconstruction → Error below 1%
4. Fast Spatial Reconstructions → Times below 1 s

METHODOLOGICAL IMPROVEMENTS

1. Vector Empirical Interpolation
2. Coefficient Based State Estimation
3. Effective Indirect Reconstruction

Future Developments

INSTRUMENTS SIMULATION AND DOMAIN INSTRUMENTATION CONSTRAINS

SENSITIVITY ANALYSIS ON THE INSTRUMENTS LOCATION AND ON THE NOISE

VALIDATION OF THE METHOD ON REAL EXPERIMENTAL DATA

TIME FILTERING STATE ESTIMATION

Thank you!